LITERATURE REVIEW: Build and Travel KD-Tree with CUDA

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1 Introduction

Ray tracing is an important and widely used tool in computer graphic. Entertainment and game industry have already benefit a lot from ray tracing. However, designers and end-users are forced to use off-line ray tracing tools for a long time due to the high computation load.

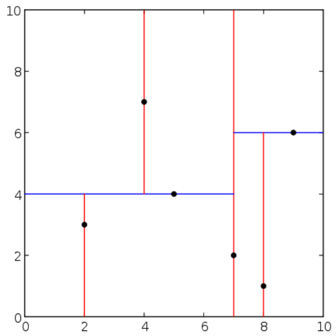
In ray tracing, most of the computation is concentrated on whether hundreds of millions of rays hit objects in a scene. The naive algorithm calculate intersection between every ray and every triangle in the scene, which yields $O(nm)$ computation time with n rays and m triangles. Inspired by binary search in one dimension space, one of the most popular and efficient improvement is partition the scene and build a tree to represent it. With this tree we have algorithm with $O(nlog(m))$. Travelling kd-tree with n rays is a natural parallel problem with each ray independent to each other. Thus if we have a kd-tree for a scene, the intersection problem can be calculated with $O(log(m))$ on n processors, which is an optimal algorithm with $nm/log(m)$ accelerator. In practice, the accelerator is much lower due to hardware limitation, which we will discuss later.

In this case kd-tree(k-dimensional tree)is developed with many other spacial partition algorithms. Despite animation with dynamic scene, kd-tree has been proved to be the most efficient data structure for static scene[2][16]. There are two tasks for us to use kd-tree in ray tracing: 1.Build kd-tree with triangle soup; 2.Travel kd-tree to find intersection points. This paper will first review state-of-the-art algorithms to build and travel kd-tree, both serial and parallel. Then implement an algorithm to build kd-tree with CUDA. Finally analyse this algorithm and try to make it faster and more memory efficient.

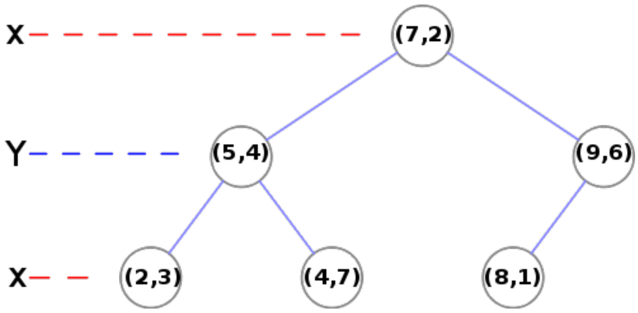
2 Literature Review

In this section, a brief introduction to kd-tree is first provided. Then some sequential algorithm will be introduced, followed by parallel algorithms, both on building and travelling kd-tree.

A kd-tree is a space-partitioning data structure for organizing points in a k-dimensional space. It is a binary tree in which every node is a k-dimensional point. Every non-leaf node can be thought of as implicitly generating a splitting hyperplane that divides the space into



(a) k-d tree decomposition for the point set (2,3), (5,4), (9,6), (4,7), (8,1), (7,2).



(b) The resulting k-d tree.

Figure 1: 2D kd-tree partition

two parts, known as half-spaces. Points to the left of this hyperplane are represented by the left subtree of that node and points right of the hyperplane are represented by the right subtree[14]. The hyperplane is perpendicular to axes, which makes kd-tree different from BSP(binary space partition) tree. Figure 1 illustrates a simple example of 2D kd-tree.

2.1 On building kd-tree

In the following, we first give the general algorithm of build a kd-tree[13], which is Algorithm 1. Consider $\mathcal{S}$ made up of N triangles. A kd-tree over $\mathcal{S}$ is a binary tree that recursively subdivides the space covered by S:The root corresponds to the axis-aligned bounding box(AABB) of $\mathcal{S}$;interior nodes represent planes $p_{k,\zeta} : x_k = \zeta$ that recursively subdivide space perpendicular to the coordinate axis;leaf nodes store references to all the triangles overlapping the corresponding voxel. Obviously, the structure of a given kd-tree(where exactly the planes are placed) directly influences how many traversal steps and triangles intersections the ray tracer has to perform. The problem is simple, how to find a "good" plane p to split V.

There are basically two kinds of strategies to choose p:

1. Median splitting[8]. To be specific, consider triangle soup in 3-dimensional space. We choose one of the three coordinates(x, y, z) each time in a circle, once selected we ignore the other two until next level. Each triangle's AABB has two projection on the current coordinate(which is also referred as events or candidates). Each time we need a p, we choose the median coordinate, and split the current triangle set into two subsets, which is the left and right child of the current node on kd-tree. We perform the same operation on the next level until the number of triangle is below threshold. We can run this algorithm in $O(nlogn)$ time, but the traversal performance is bad, because the median split strategy does not take triangle distribution into account.

2. Heuristic search[6][13][10][9]. Instead of split current node from the median point, heuristic algorithm search for the best p in the solution space. We can place p on any place between AABB bounding of current node. The best p gives the lowest travel cost on the kd-tree rooted by current node. The cost function $cost(x)$[3] is defined as

Algorithm 1 Recursive KD-tree build

```
if Terminate(T, V) then
return new leaf node(T)
function RecBuild(triangles T, voxel V)return node
    if Terminate(T, V) then
        return new leaf node(T)
    end if
    p = FindPlane(T, V) Find a good plane p to split V
    (V_L, V_R) = Split V with p
    T_L = {t ∈ T|(t ∩ V_L) ≠ ∅
    T_R = {t ∈ T|(t ∩ V_R) ≠ ∅
    return new node(p, RecBuild(T_L, V_L), RecBuild(T_R, V_R))
end function

function BuildKDTree(triangles[] T))return root node
    V = B(T){start with full scene}
    return RecBuild(T, V)
end function
```

follows:

$$cost(x) = C_I + C_L(x)\frac{SA_L(t,x)}{SA(t)} + C_R(x)\frac{SA_R(t,x)}{SA(t)} \tag{1}$$

Where C_I is the cost of traversing the node itself, $C_L(t)$ is the cost of the left child given a split position x and $C_R(x)$ is the cost of the right child given the same split. $SA_L(t,x)$ and $SA_R(t,x)$ are the surface areas of the left and right children respectively. $SA(v)$ is the surface area of the triangle currently being considered for splitting. We can approximate $C_L(x)$ and $C_R(x)$ by just count number of triangles in the left and right sub-tree respectively. This evaluation is also called surface area heuristic(SAH) and widely used not only by kd-tree but also by other spatial partition algorithms such as bounding volume hierarchy(BVH), which has been proved to be faster than kd-tree in dynamic scenes[16][9].

In order to calculate number of triangles line on the left and right side of the plane at each x, we can iterate over all candidates to see if it belongs to the left or the right. This takes$O(n^2)$ time for n candidates. It is obvious that $O(n^2)$ algorithms are impractical except for trivially small n. We can get immediate speed up by sort t first at a cost of $O(nlogn)$. And then sweep t again to calculate $C_L(x)$ and $C_R(x)$ at a cost of $O(n)$. This algorithm results in $O(nlog^2n)$ for the kd-tree as a whole.

Wald has proved that the lower bound for construction of a kd-tree is $O(nlogn)$[13]. To achieve this goal, researchers have posted many solutions.

Wald himself describe an optimal sequential $O(nlogn)$ SAH kd-trss construction algorithm[13] that initially sorts the candidates extents in the thress coordinate axes, performs linear-time sorted-order coordinate sweeps to compute the SAH just like the $O(nlog^2n)$ method while maintains sorted order as the bounding boxes and their constituent geometries are moved and subdivided. It actually maintains 3 sorted array for 3 axes. When p is chosen, it puts left and right child to new sorted array for 3 axes individually, and sort all the candidates that cross p. Since the maximum number of triangles across p is no larger than$\sqrt{n}$ [13][10], sorting them takes $O(\sqrt{n}log\sqrt{n}) < O(n)$, so the overall cost is $O(nlogn)$.

To allow a tradeoff between tree quality and construction speed, Hunt[6] gives an approximation of equation 1. Instead of calculate all the possible p, they calculate only 16 samples, then use piecewise quadratic function approximately find the lowest $cost(x)$ for p. They proved that the error is bounded.

Leaving the *cost* function alone, Popov play with the construction function on a lower level[10]. Popov's approach is based on Wald's theory but exploit memory usage. They try to use BFS to construct kd-tree instead of DFS, thus yield high local memory visit and memory coherence. But not all of their algorithm runs in BFS. On the lower level of kd-tree they still use DFS. We know that DFS is p-complete problem and thus not easy to be paralleled.

We have talked about build SAH kd-tree sequentially, now we are moving to the parallel situation. First Shevtsov developed a parallel kd-tree construction approach on multi-core CPU[12]. They divide the whole scene into several sets that have almost the same number of triangles, and then process them independently in parallel. Another idea they come up to is the Max-Min binning scheme. Instead of computing the precise SAH, they approximate the SAH at some equally sized bins. But this approach is not suit for GPU because GPU has thousands of threads while cutting the scene in such manner will digress the optimal solution.

Zhou developed the first kd-tree construction runnning entirely on GPU[16]. They classify the nodes into large nodes and small nodes by the number of triangles in the nodes or, on the other hand, level of nodes in the tree. They use median split for large nodes and SAH for small nodes. Also they use tight data structure to achieve high local memory hit. Their result shows that their algorithm is outstanding. However, large nodes in this algorithm may degrade the kd-tree quality if there are too many of them.

Choi[1] and Wu[15] present algorithms on GPU without degrade the quality of kd-tree respectively. Wu use GPU scan tto count triangle number of the child nodes and a bucket-based algorithm to sort the AABBs.

2.2 On travel kd-tree

For the sequential travel algorithm, one can refer to [2]. The idea is simple,The algorithm takes as input a tree and a ray, and searches for the rest primitive in the tree that is intersected by the ray. The tree is traversed starting at the root, and a stack is used as a priority-ordered list of nodes left to visit. Each node on the stack is closer to the ray origin than all nodes below it, and the node currently being traversed is closer than all nodes on the stack. A (tmin; tmax) range limits the part of the ray under consideration to that which intersects the current node.

When an internal node is encountered during the traversal, the(tmin,tmax)range of the ray is classified with respect to the splitting plane of the node. If the range lies entirely to one side of the plane, the traversal simply moves to the appropriate child. If, instead, the range straddles the plane then traversal will continue to the rest child hit by the ray, while the second child is pushed onto the stack along with its appropriate(tmin,tmax)range. In this way the traversal proceeds down the tree, occasionally pushing items onto the stack, until a leaf node is reached.

If the ray intersects one of the primitives in the leaf within the(tmin,tmax)range, then the closest intersection in the leaf is guaranteed to be the rest intersection along the ray, and the traversal terminates and yields this result. If no intersection is found then we pop a work item consisting of a node and a (tmin, tmax) range from the stack and continue searching. If the stack is empty then there is no intersection along the ray and the search terminates.

The main challenge for parallel traversal kd-tree is eliminate the stack operation. GPU is not good at stack operations mainly because the local memory is too small. If a stack operation needs to visit global memory, it can be very slow. Fortunately there are already kd-tree traversal algorithms that do not need stack, and the most basic ones are KD-Restart and KD-Backtrack[2].Horn[4] and Popov[11] also provide stack-less kd-tree traversal algorithm.

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